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THEORETICAL REALIZATION OF THE STIRLING CYCLE ON TWO PISTONS MACHINES

BY

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Abstract: The paper defines the theoretical Stirling machine, that has heat exchangers with null volume, represented by portions of the cylinder. The paper presents the realization of the Stirling cycle on alpha, beta and gamma Stirling motors. The pressure-volume diagrams inside the machine chambers are obtained. For Stirling motors with identical maximum compression chamber volumes, the thermodynamic inferiority of the gamma scheme is proved.

Key words: theoretical Stirling machine, Stirling cycle, p-V diagram, compression and expansion chambers

1. Introduction

The realization of the Stirling thermodynamic cycle with heat regeneration is presented in the literature [1], [2], [3] in connection with a model engine having two pistons that delimit the compression and the expansion chambers in either one cylinder or two cylinders. The motor is fitted with a heat regenerator, and possibly with a heater and a cooler. All heat exchangers are placed outside the cylinder (fig. 1 a).

Inside a model Stirling engine like the one in fig. 1 the gas cannot perform a Stirling cycle, because the two needed isotherms cannot be achieved (a mass of gas being trapped in the regenerator, at a temperature different than the ones of the heat sources).

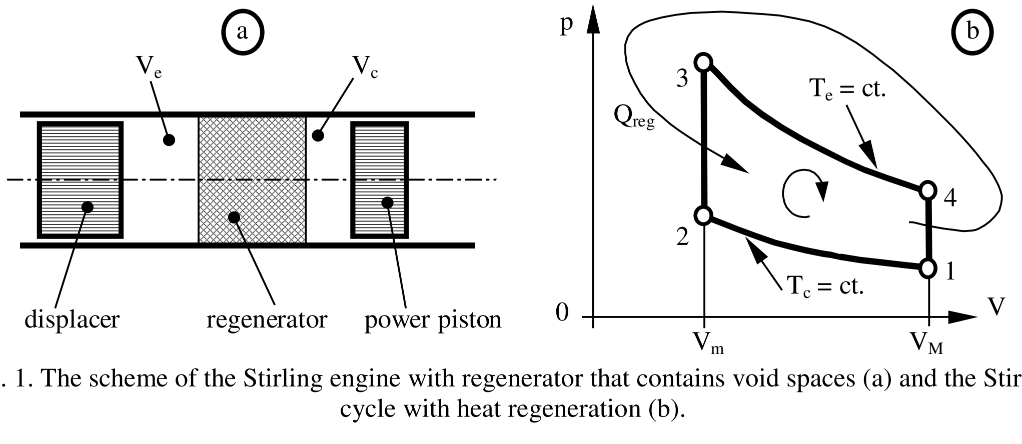

Fig. 1. The scheme of the Stirling engine with regenerator that contains void spaces (a) and the Stirling cycle with heat regeneration (b).

The paper defines rigorously the theoretical Stirling engine and analyzes the way in which the thermodynamic Stirling cycle is realized in this machine. The theoretical model of the machine is applied to the constructive variants alpha, beta and gamma.

2. Defining the Theoretical Stirling Machine

The theoretical Stirling machine is a machine in which the working agent performs the thermodynamic Stirling cycle (fig. 1 b).

The compression isotherm 1-2 takes place at the cycle minimum temperature, so that it is impossible to have gas amounts with higher temperatures inside other chambers. As a consequence, the volume of the regenerator must be zero. In the compression period the expansion chamber must also have null volume, and the displacer must halt near the cylinder head.

The expansion isotherm 3-4 takes place at the cycle maximum temperature, so that it is impossible to have gas amounts with lower temperatures inside other chambers. As a consequence, during the expansion period the volume of the compression chamber must be null, so that either the power piston is halted near the cylinder head (at the alpha type machines), or the power piston moves simultaneously with the displacer (at beta type machines).

From the two previously enunciated conditions it results that the theoretical Stirling engine must have only two chambers, one for compression and one for expansion.

In order to maintain constant the total volume occupied by the gas inside the machine (in the processes 2-3 and 4-1), the power piston must halt (at beta and gamma schemes) or the pistons must have interrelated movements (at alpha scheme).

The realization of the isochoric process 4-1 means that the maximum volume of the expansion chamber must be equal to the maximum volume of the compression chamber. At the alpha scheme, the trivial solution that responds to this condition is to use pistons with the same diameter and the same stroke. If the pistons have stems, their diameter must be negligible.

The machine chambers must maintain their functional role over the entire period of the cycle. As a consequence, inside the compression and expansion chambers processes at constant temperature can take place only.

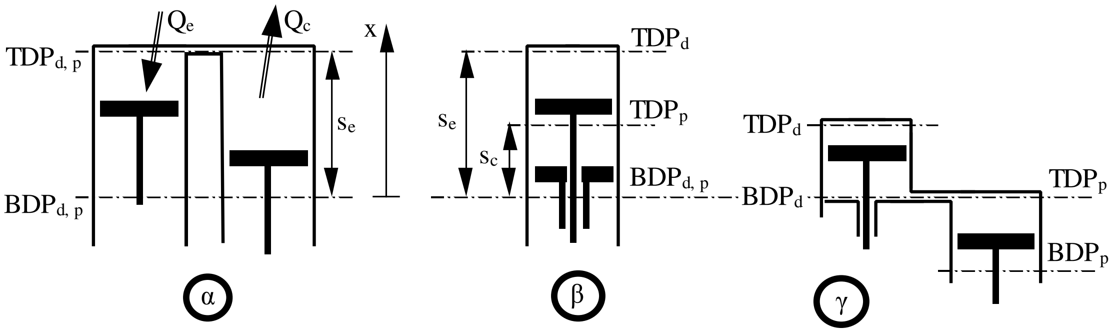

Fig. 2. Constructive schemes for theoretical Stirling machines.

Inside the theoretical Stirling machine the heat transfer is realized through portions of the cylinder and of the cylinder head. The regenerator is represented also by a portion of the cylinder walls.

Inside the theoretical Stirling machine a constant mass of ideal gas evolves. The pressure losses cannot occur and the instantaneous pressure is the same inside all the machine chambers.

The duration of the functional phases are arbitrary, because at the theoretical machine the performances are given by the law of variation of pressure as function of volume and not of time.

The piston movement laws are also arbitrary, for the same reason: the performances depend only by the p(V) variation law. The periods during which the pistons remain stationary and the periods during which the pistons have correlated movements depend on the constructive scheme of the machine (fig. 2).

In conclusion, inside the theoretical Stirling machine a constant mass of working gas evolves inside two isothermal chambers. The chamber volumes vary between zero and identical maximum values. The machine exchanges heat with the surroundings through the cylinder walls.

3. Theoretical Stirling Machine Performances

The variation of the instantaneous pressure inside the theoretical machine is obtained from the equation of the conservation of the mass inside the machine:

$$m_T = m_c + m_e = \frac{p\,V_c}{R\,T_c} + \frac{p\,V_e}{R\,T_e} \tag{1}$$

and has the following expression

$$p = \frac{m_T\,R}{\dfrac{V_c}{T_c} + \dfrac{V_e}{T_e}}, \tag{2}$$

where R is the individual gas constant and the subscript T means total.

The works exchanged inside the chambers and inside the machine are obtained with the defining relation:

$$L = \oint p\,dV\,. \tag{3}$$

From the first law of the thermodynamics we obtain that, for either the compression or expansion chamber, during one complete cycle, the heat exchanged is equal to the work exchanged.

The efficiency of the motor is equal with the efficiency of the Carnot cycle evolving between the same temperatures (due to the ideal heat regeneration) and can be calculated with the relation:

$$\eta = 1 + \frac{Q_c}{Q_e} = 1 + \frac{L_c}{L_e}\,. \tag{4}$$

The heat Q_c is removed from the cycle and the work L_c is spent, so their values are negative.

4. Theoretical Realization of the Stirling Cycle on Alpha-Type Motors

The analysis of the realization of the Stirling cycle on theoretical machines must allow the comparison of the performances that could be obtained by various constructive type machines. The analyzed machines fulfil, besides the requests displayed in chapter 2, the following conditions:

- the displacer and the power piston diameters are equal;
- the piston strokes are chosen in order to obtain equal maximum volumes for the compression chamber;
- the duration of the functional phases is the same;
- the piston movement laws are linear;
- the compression ratio $V_M / V_m = 2$ (chosen);
- the same mass $m_T = 0.0025$ kg of hydrogen evolves between the same temperatures $T_c = 310$ K and $T_e = 773$ K.

For a theoretical alpha-type Stirling engine (fig. 2 α) the pistons strokes are equal (fig. 3 a), the power piston standing still during the expansion process and the displacer standing still during the compression process.

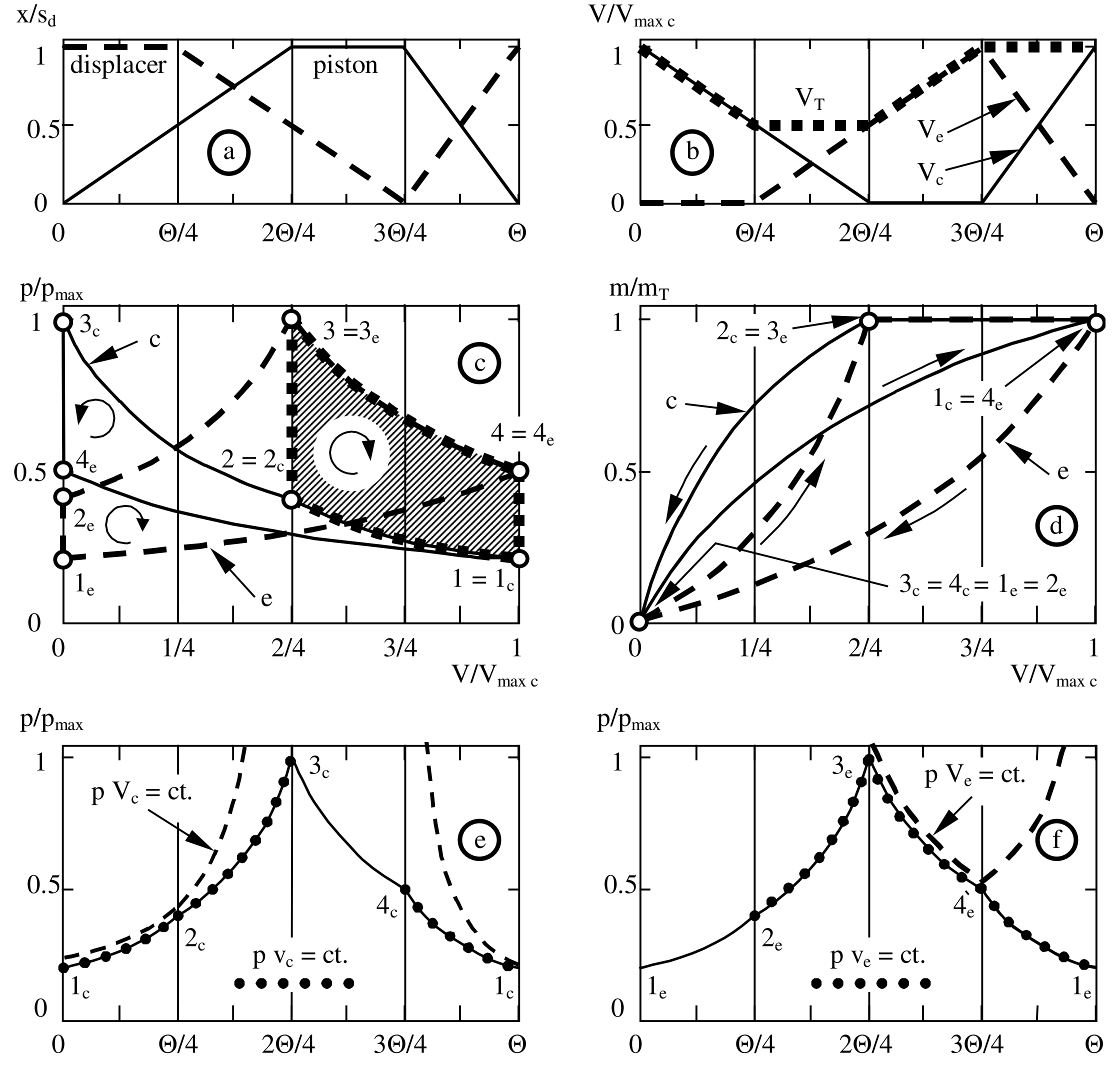

Fig. 3. Theoretical realization of the Stirling cycle on an alpha-type motor: a - piston movement laws; b - volume variation laws; c - thermodynamic cycles inside the chambers and inside the motor; d - the variation of the gas masses inside the chambers; e and f - isotherms inside the machine chambers; x - piston actual position; s_d = displacer stroke; $V_{max\ c}$ - maximum volume of the compression chamber; p_{max} - cycle maximum pressure; Θ - cycle period.

Inside the compression chamber the following processes take place: the isothermal compression (with constant mass m_T - the process 1_c-2_c on fig. 3c), the isothermal processes of gas exchange with the expansion chamber (with variable mass,

2_c-3_c and 4_c-1_c) and the conventional process 3_c-4_c. The conventional process 3_c-4_c takes place with zero mass of gas and at zero volume. It corresponds to the process during which the gas inside the compression chamber leaps from the state 3_c (characteristic to the end of the gas displacement into the expansion chamber) to the state 4_c (characteristic to the beginning of the gas displacement back to the compression chamber).

Inside the expansion chamber the following processes take place: the isothermal expansion (with constant mass m_T - the process 3_e-4_e on fig. 3c), the isothermal processes of gas exchange with the compression chamber (with variable mass, 2_e-3_e and 4_e-1_e) and the conventional process 1_e-2_e. The conventional process 1_e-2_e takes place with zero mass of gas and at zero volume. It corresponds to the process during which the gas inside the expansion chamber leaps from the state 1_e (characteristic to the end of the gas displacement into the compression chamber) to the state 2_e-4_e (characteristic to the beginning of the gas displacement back to the expansion chamber).

The processes inside the whole theoretical Stirling machine must be obtained by summing the processes that take place inside the compression and expansion chambers. During the compression and expansion processes all the gas lies inside either the compression chamber or the expansion chamber, and the constant mass isothermal processes 1-2 and 3-4 are obtained.

The process of gas transfer from one chamber to another (the processes 2-3 and 4-1) take place through the correlated variation of the chamber volumes, while the total volume occupied by the gas remains constant. In each chamber isothermal processes with variable mass take place, but inside the machine as a whole the temperature varies accordingly to the equation of the isochore.

On fig. 3d the processes that take place with constant mass and also the processes of gas transfer can be observed.

In fig. 3e and f the isothermal processes with constant mass 1_c-2_c and 3_e-4_e ($p\ V$ = const., and simultaneously $p\ v$ = const.) that take place inside the compression and expansion chambers during the respective functional phases, and the isothermal processes with variable mass ($p\ v$ = const.) that take place in the machine chambers during the gas transfer from one chamber to another are emphasized.

5. Theoretical Realization of the Stirling Cycle on Beta-Type Motors

For a theoretical beta-type Stirling motor (fig. 2 β) the power piston stroke must be shorter than the displacer stroke. On fig. 4a we have $s_c = s_e/2$, in order to fulfil the chosen compression ratio, and the power piston stands still during the gas transfer phases. The gas transfers are obtained only by the displacer movements.

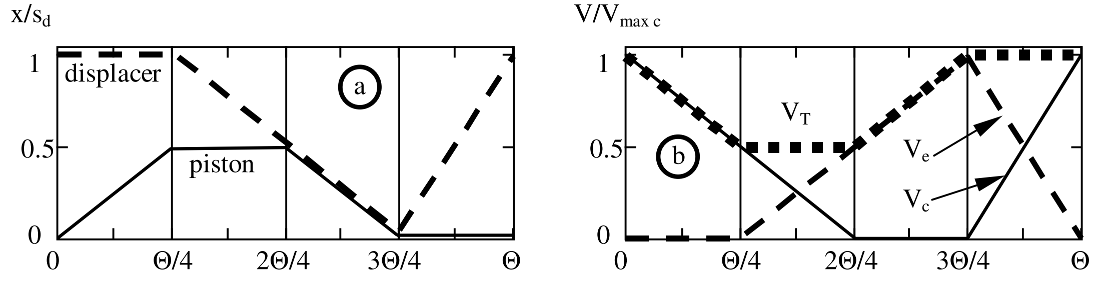

Fig. 4. Theoretical realization of the Stirling cycle on a beta-type motor:
a - piston movement laws; b - volume variation laws.

The chamber volume variation laws (fig. 4 b) are identical with the ones obtained for the theoretical alpha-type machine. So, the thermodynamic cycles inside the machine and inside the machine chambers are identical with the ones obtained for the theoretical alpha-type machine (fig. 3 c and d).

6. Theoretical Realization of the Cycle on Gamma-Type Motors

For a theoretical gamma-type Stirling engine (fig. 2 γ) the condition $V_{\max c}$ = const. requires equal piston strokes. On fig. 5 a, the strokes are half of the beta or alpha displacer stroke, in order to obtain the same compression ratio. The power piston stands still during the gas transfer phases and the displacer stands still during the compression and the expansion phases.

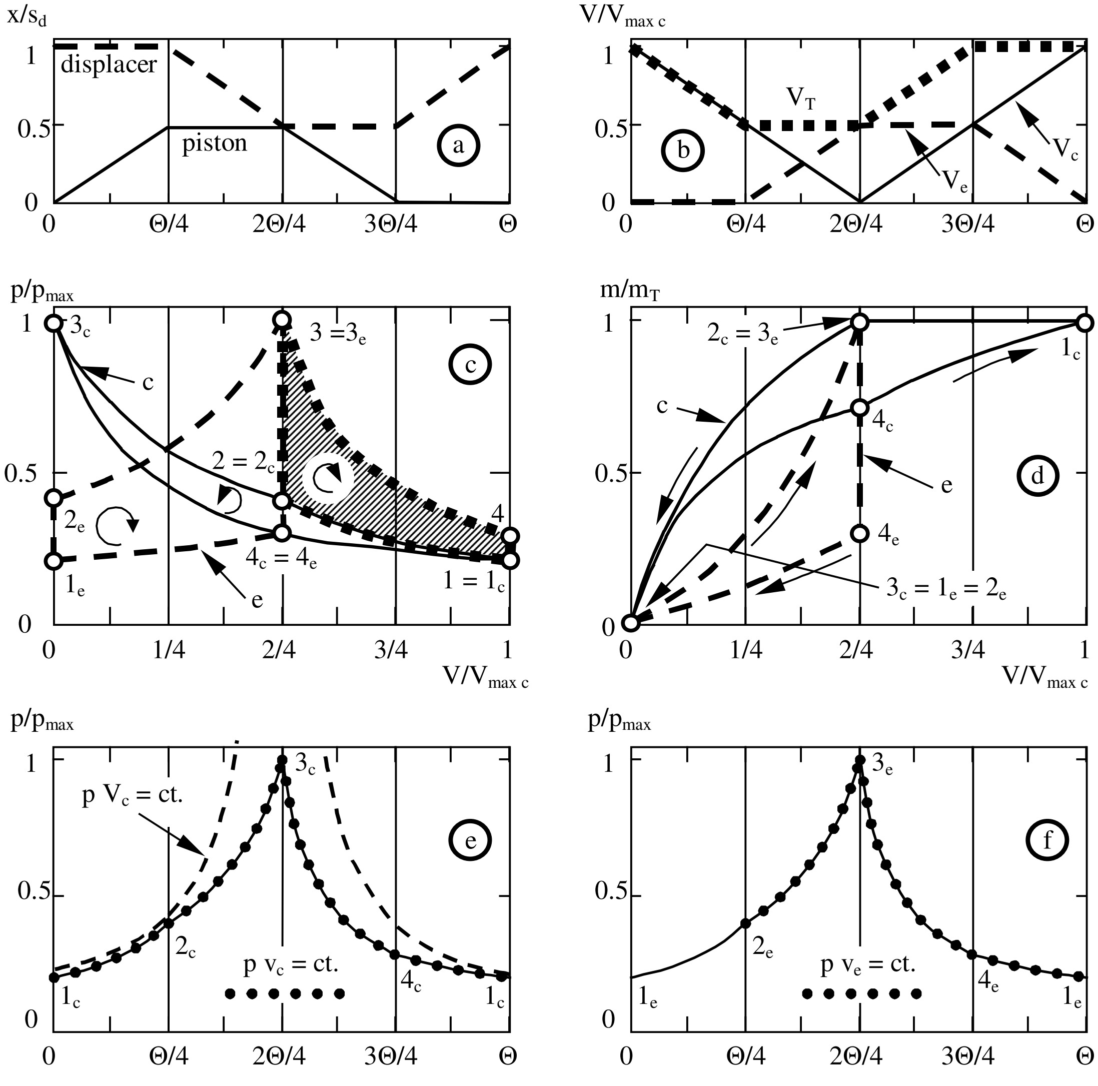

Fig. 5. Theoretical realization of the functioning cycle on a gamma-type Stirling motor: a - piston movement laws; b - volume variation laws; c - thermodynamic cycles inside the chambers and inside the motor; d - the variation of the gas masses inside the chambers; e and f - isotherms inside the machine chambers.

The analysis of the functioning, described by the curves on fig. 5, shows that inside the gamma-type machine only one process with constant mass takes place, i.e. the compression. The expansion 3-4 is a process with variable mass, the gas lying partially inside the expansion chamber and partially inside the compression chamber.

The expansion process for the whole machine 3-4 is no longer isothermal and so inside the gamma-type machine the gas cannot evolve in a Stirling cycle (composed from two isotherms connected by two isochores).

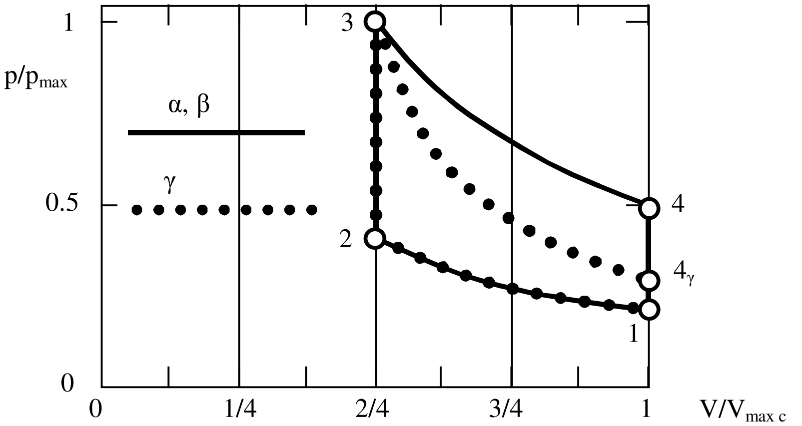

Fig. 6. Comparison between the cycles realized in theoretical alpha, beta and gamma Stirling motors.

The comparison between the cycles realized in theoretical alpha, beta and gamma motors is presented in fig. 6 and shows that in alpha and beta motors the gas evolves in a Stirling thermodynamic cycle. In gamma motors the gas evolves in a cycle that differs from the Stirling cycle and provides inferior performances. For all the compared motors the agent mass, the maximum volume of the compression chamber, the compression ratio and the temperature levels are the same.

7. Conclusions

The paper analyzes the requirements that must be accomplished by a gas thermal machine with two pistons in order that the gas evolve in a Stirling thermodynamic cycle.

The theoretical Stirling machine was defined. In this machine a constant mass of ideal gas evolves, lying inside two isothermal chambers. The compression and expansion volumes vary in correlation between zero and equal maximum values. The machine exchanges heat through the cylinder walls.

It was proved that inside the alpha- and beta-type theoretical machines the gas can evolve in Stirling thermodynamic cycle. These machines accomplish one isothermal compression process and one isothermal expansion process, both with constant mass. The processes take place between the same extreme values of the volume.

It was proved that the expansion process that takes place inside the gamma machine cannot be an isothermal one and as a consequence inside the gamma-type machine the gas cannot evolve in a Stirling cycle. So, the gamma-type machine is not a Stirling machine in the sense given by the definition proposed in the paper for the theoretical Stirling machine.

Inside the gamma-type machine the gas evolves in a thermodynamic cycle composed from an isothermal compression with constant mass, two isochoric processes of gas transfer from one chamber to another and from one expansion process that is not a basic thermodynamic process.

For theoretical machines having the same mass of gas, the same maximum volume of the compression chamber, the same compression ratio and the same temperature levels the comparison between the cycles realized on alpha-, beta- and gamma-type machines shows that the gamma motor has performances inferior to the other constructive schemes.

The analysis of the theoretical Stirling machines allows a better understanding of their functioning and of their maximum performances.

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REALIZAREA TEORETICĂ A CICLULUI STIRLING PE MAŞINI CU DOUĂ PISTOANE

(Rezumat)

Este definită maşina Stirling teoretică, la care schimbătoarele de căldură nu au volum propriu, fiind materializate de porţiuni de cilindru. Este prezentată realizarea ciclului Stirling pe maşini teoretice de tip alfa, beta şi gama. Sunt stabilite diagramele presiune-volum din camerele funcţionale. Pentru maşini cu camere cu volume maxime egale, aflate în aceleaşi condiţii de funcţionare, este dovedită inferioritatea termodinamică a schemei constructive de tip gama.